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Testing Bell inequalities in photonic crystals

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Abstract. We show how entangled atomic pairs can be prepared in order to test the Bell inequalities. The scheme is based on the interaction of the atoms with a highly localized field mode within a photonic crystal. The potential of using optically separated transitions and the stability of the entangled state to spontaneous emission could lead to the closure of the communication and the detection loopholes appearing in experiments so far. The robustness of the scheme against detector inefficiencies, the spread in the atomic velocities and the fact that the entangled pairs are not generated simultaneously is also studied.

PACS. 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.) – 42.50.-p Quantum optics – 42.70.Qs Photonic bandgap materials

During the past twenty years, a number of interesting experiments for testing the Bell inequalities [1,2] have been proposed and carried out. In most, the entangled particles are photons, [3,5,8]. Violation of the inequalities occurs, supporting the quantum mechanical description of Nature against local realistic theories. However in all these photon experiments, low detection efficiency combined with low analyzing speed of the results prevented the complete closure of the detection and/or the communication loophole. Experiments [8] which partially or even completely close one of the loopholes exist, *e.g.* the experiment by Weihs *et al.* [9] where the communication loophole was closed. However no photon experiment closing both loopholes has been reported so far.

In addition to photon based experiments there have also been proposals using atoms, based on the direct manipulation of the atomic degrees of freedom, by interaction with the quantized field of the micromaser [10–13] or by photo-dissociation of dimers [4,15] and more recently by laser manipulating ions trapped inside a cavity [16]. Although in these elegant experiments the detection efficiency was higher than the photon based ones, problems such as as sequential detection (for the micromaser based experiments) and fragility of the Rydberg atoms at room temperature did not allow the complete closure of all the loopholes.

We propose to create entangled atomic states using a photonic crystal (or photonic band gap material-PBG) for a Bell inequalities test. The potential of using optically separated transitions which can be detected very efficiently, the stability against background radiation, and the inhibition of spontaneous emission inside the crystal can lead to the closure of all loopholes appearing in previous experiments.

In our scheme the entanglement originates from the interaction of two atoms with a resonant defect mode inside a photonic crystal. Photonic crystals are highly porous three dimensional periodic materials of high refractive index with pore periodicity on the length scale of the relevant wavelength of light. They exclude electromagnetic modes over a continuous range of frequencies [17,18,20,25] (a photonic band gap). By introducing voids that are larger than the rest of the array, strongly localized single modes of light can be engineered within the otherwise optically empty PBG [19].

Our system consists of two two level atoms, the first of which is initially prepared in the upper of two optically separated states, denoted by $|e_1\rangle$ and the second in the lower one $|g_2\rangle$ [10,11]. The two atoms propagate sequentially in orthogonal directions through the defect region of the crystal (see Fig. 1). The defect mode is initially prepared in the vacuum state $|0\rangle$ and it is on resonance with the atomic transition $|e_\mathrm{i}\rangle \rightarrow |g_\mathrm{i}\rangle$ (Fig. 2). Although our scheme has some similarities with those proposed by [10,11] and implemented by [13], the potential of entangling and manipulating conventional, rather that Rydberg atoms in a spontaneous emission free environment (the photonic crystal) opens the way to a new class of loophole free Bell experiments.

The dynamics of a two level atom passing through a point defect [21–23], under the dipole and rotating wave approximations are described by the Jaynes-Cummings Hamiltonian [7,21]:

$$H(\mathbf{r}) = \frac{\hbar\omega_\mathrm{a}}{2}\sigma_z + \hbar\omega_\mathrm{d} a^\dagger a + \hbar G(\mathbf{r})(a\sigma_+ + a^\dagger\sigma_-) \;, \quad (1)$$

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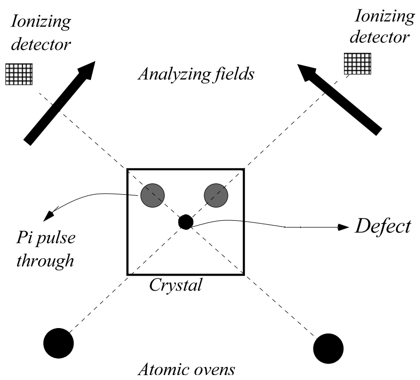

Fig. 1. The proposed scheme for creating entangled atomic pairs using a photonic crystal.

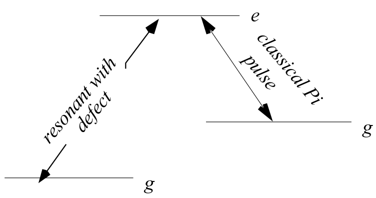

Fig. 2. The atomic system consideration. The transition $|g\rangle \rightarrow |e\rangle$ is resonant with the defect frequency whereas the $|e\rangle \rightarrow |g'\rangle$ undergoes a Π rotation just before the atoms exit the crystal. The latter is possibe through the coupling with an external laser field injected in the crystal through a properly engineered line defect.

where σ_z, $\sigma_\pm = \sigma_x \pm \sigma_y$ are the Pauli spin operators for a two-level atom with transition frequency $\omega_{\rm a}$, and a, $a^\dagger$ are the annihilation and creation operators for a photon in a defect mode of frequency $\omega_{\rm d}$. The atom field coupling strength may be expressed as $G(\mathbf{r}) = \Omega_0\big(\hat{\mathbf{d}}_{\rm eg}\cdot\hat{\mathbf{e}}(\mathbf{r})\big)f(\mathbf{r})$, where Ω_0 is the peak atomic Rabi frequency over the defect mode, $\hat{\mathbf{d}}_{\rm eg}$ is the orientation of the atomic dipole moment, and $\hat{\mathbf{e}}(\mathbf{r})$ is the direction of the electric field vector at the position of the atom. The three-dimensional mode structure is a function of the size and shape of the defect. In our case we need only consider the one-dimensional mode profile that intersects the atom's linear path. The profile $f(r)$ has an exponential envelope centered about the point in the atom's trajectory that is nearest to the center of the defect mode, r_0. Within this envelope, the field intensity oscillates sinusoidally, and for fixed dipole orientation, variations in the relative orientation of the dipole and the electric field gives a sinusoidal contribution.

In our case we set $\hat{\mathbf{d}}_{\rm eg}\cdot\hat{\mathbf{e}}(\mathbf{r}) = 1$ and write

$$f(r) = {\rm e}^{-\frac{|r-r_0|}{R_{\rm def}}}\cos\left[\frac{\pi}{a}(r-r_0)+\phi\right]\cdot \tag{2}$$

Also $R_{\rm def}$ defines the spatial extent of the mode which is at most a few lattice constants for a strongly confined mode in a PBG, and $\phi = 0$.

The atom-field state function after an initially excited atom has passed through a defect can be written as

$$|\Psi\rangle = C_{\rm e}(t_1)|{\rm e}\rangle|0\rangle + C_{\rm g}(t_1)|g\rangle|1\rangle, \tag{3}$$

where $C_{\rm e/g}$ are the amplitudes of the atom being in the excited/ground atomic state and t_1 is the interaction time of atom 1 with the defect. As soon as atom 1 leaves the defect region, atom 2 in its ground state is sent through. Letting t_2 be its interaction time and taking into account that $t_{\rm i} = v_{\rm i}/R_{\rm def}$ the final state of the system can be written as follows

$$\begin{aligned}|\Psi_{\rm f}\rangle = C_{\rm e}(v_1)|e_1\rangle|g_2\rangle|0\rangle + C_{\rm g}(v_1)C_{\rm e}(v_2)|g_1\rangle|g_2\rangle|1\rangle \\ + C_{\rm g}(v_1)C_{\rm g}(v_2)|g_1\rangle|e_2\rangle|0\rangle\cdot\end{aligned} \tag{4}$$

Following the approach of references [21–23], we solved numerically the above equation for the $\lambda = 780$ nm transition of an initially excited Rb atom. We assumed that the atom travels through a point defect in a optical photonic crystal [24] at thermally-accesible velocities ($v \sim 100$–600 m/s) and is on resonance with the defect mode ($\omega_{\rm a} = \omega_{\rm d} = 2.4\times10^{15}$ rad/s) (see Fig. 1). The rest of the parameters are $R_{\rm def} = a$, $\phi = 0$, $a = 0.8\lambda$, and $\Omega_0 = 1.1\times10^{10}$ rad/s [22]. The singlet state is produced for values of the atomic velocities equal to $v_1 = 231$ m/s and $v_2 = 270$ m/s as for them $C_{\rm e}(v_2) = 0$, $C_{\rm g}(v_2) = 1$ and $C_{\rm e}(v_1) = -C_{\rm g}(v_1) = 1/\sqrt{2}$. If we let v_1, v_2 be arbitrary we then obtain the density matrix

$$\begin{aligned}\rho &= {\rm Tr}_{\rm field}[|\Psi_{\rm f}\rangle\langle\Psi_{\rm f}|] \\ &= \rho_1|e_1,g_2\rangle\langle e_1,g_2| + \rho_2|g_1,g_2\rangle\langle g_1,g_2| \\ &\quad +\rho_3|g_1,e_2\rangle\langle g_1,e_2| + \rho_4|e_1,g_2\rangle\langle g_1,e_2| \\ &\quad +\rho_5|g_1,e_2\rangle\langle e_1,g_2|\end{aligned} \tag{5}$$

for the two atoms. The coefficients

$$\begin{aligned}\rho_1 &= |C_{\rm e}(v_1)|^2, \\ \rho_2 &= |C_{\rm g}(v_1)C_{\rm e}(v_2)|^2, \\ \rho_3 &= |C_{\rm g}(v_1)C_{\rm g}(v_2)|^2, \\ \rho_4 &= C_{\rm e}(v_1)C^*_{\rm g}(v_1)C^*_{\rm g}(v_2), \\ \rho_5 &= C_{\rm g}(v_1)C_{\rm g}(v_2)C^*_{\rm e}(v_1)\end{aligned} \tag{6}$$

obey the normalization condition $\rho_1+\rho_2+\rho_3 = 1$.

Test Bell's inequality we measure the quantity [1]

$$\mathcal{S} = E(\phi,\theta) - E(\phi,\theta') + E(\phi',\theta) + E(\phi',\theta') \tag{7}$$

where $E(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\theta}})$ is the expectation value of $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ along the directions $\hat{\boldsymbol{\phi}}$, $\hat{\boldsymbol{\theta}}$

$$E(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\theta}}) = \langle\boldsymbol{\sigma}_1\cdot\hat{\boldsymbol{\phi}}\ \boldsymbol{\sigma}_2\cdot\hat{\boldsymbol{\theta}}\rangle = {\rm Tr}(\rho\boldsymbol{\sigma}_1(\hat{\boldsymbol{\phi}})\boldsymbol{\sigma}_2(\hat{\boldsymbol{\theta}})) \tag{8}$$

with $\boldsymbol{\sigma}_i = (\sigma_{ix},\sigma_{iy},\sigma_{iz})$ being the Pauli spin operator for the two level atom i. If $\hat{\theta} = (\sin\alpha, 0, \cos\alpha)$ and $\hat{\theta} = (\sin\beta, 0, \cos\beta)$, then

$$E(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\theta}}) = (2\rho_2-1)\cos\alpha\cos\beta + (\rho_4+\rho_5)\sin\alpha\sin\beta. \tag{9}$$

For the special case of $v_1 = 231$ m/s and $v_2 = 270$ m/s, *i.e.*, the singlet state, this reduces to

$$E(\hat{\boldsymbol{\phi}},\hat{\boldsymbol{\theta}}) = -\cos(\alpha-\beta) \tag{10}$$

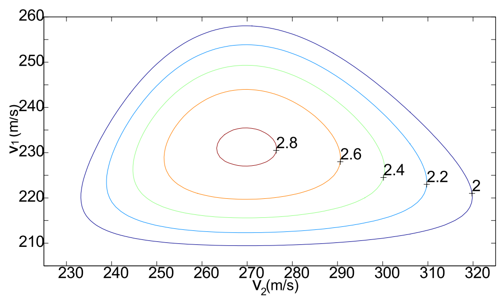

Fig. 3. Contour plot of Bell's parameter $\bar{\mathcal{S}}$ as function of the atomic velocities v_1, v_2.

and with $\alpha = 0, \alpha' = \pi/2, \beta = \pi/4$, and $\beta' = 3\pi/4$ we obtain from equation (7) $|\mathcal{S}| = 2\sqrt{2}$, the maximum violation of Bell's inequality.

With these choices of angles, in the more general case of arbitrary v_1, v_2, the quantity S is given by

$$S = \sqrt{2}|(2\rho_2 - 1) + (\rho_4 + \rho_5)|. \tag{11}$$

For two-level atoms, the measurement of the Pauli operators along any direction ϕ can be performed [10–14] by applying a controlled pulse to the atom which transforms the states $|e\rangle$ and $|g\rangle$ into the eigenstates of $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\phi}}$; a subsequent measurement of the state of the atom (ground or excited) would give the desired measurement of the observable corresponding to $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\phi}}$ (see Fig. 1).

In Figure 3 we show contour plots of $|S|$ *versus* the velocities. The values of the parameters correspond to the case of Rb atoms traveling through a defect in an optical photonic crystal (*e.g.*, GaP) at thermal velocities ($v \sim 150-400$ m/s). The extend of the defect mode is assumed to be equal to the lattice constant a and $a = 0.8\lambda$. The coupling constant has a maximum value of 1.1×10^{10} rad/s at the center of the defect. The dashed lines in the plot represent to $|S| = 2.2, 2.4, 2.6, 2.8$, whereas the solid line corresponds to $S = 2$, the maximum value allowed by local realistic theories.

As the transitions here are optically separated, decoherence due to spontaneous emission would potentially be a problem during the flight from the crystal to the analyzers/detectors (inside the crystal spontaneous emission is competely inhibited [18]). To tackle this problem through a properly engineered line defect [26] we inject a coherent light field into the crystal which crosses the atom's path. This field is resonant with the $|e\rangle \rightarrow |g'\rangle$ transition. By adjusting the field intensity properly we can apply a Π pulse between $|e\rangle$ and $|g'\rangle$ transferring any population from $|e\rangle$ to $|g'\rangle$ (see Fig. 2). The transition $|g'\rangle \rightarrow |g\rangle$ is dipole non-allowed and thus can be considered stable for our purposes (lifetime$\sim$1 s). At thermal velocities of the order of few hundred m/s, the minimum field strength, E, required to fully rotate the Bloch vector for the optical transition $|e\rangle \rightarrow |g'\rangle$, ($\lambda \sim 1$ μm), is of the order of few kV/m. This magnitude of field strength should be attainable using a cw laser whose output is coupled into waveguide channels of λ^2 cross-section. This is well below the ionization field strengths of both the flying atom and the photonic crystal.

Our scheme is robust against inefficiencies and problems that are usually encountered in other atomic Bell experiments. Firstly, careful attention should be paid on the constitution of the ensemble of pairs on which the Bell test is to be performed [12], due to the fact that our entangled pairs are not generated together as the photons in the corresponding atomic cascade or parametric down conversion experiments [2,3,5,6,8], More specifically, the mean times T_1 between atoms from oven 1 (1-atoms), the mean times T_2 between atoms from oven 2 (2-atoms), the average interaction times $\bar{\tau}_1$ of 1-atoms with the defect, the average interaction times $\bar{\tau}_2$ of 2-atoms with the defect, and the lifetime $\tau_{\rm def}$ of a photon in the defect should satisfy the following conditions: $T_1 > 10\tau_{\rm def}$, $\tau_{\rm def} > 100\bar{\tau}_1$, $T_2 \approx 10\bar{\tau}_1 \approx 10\bar{\tau}_2$.

Our scheme is capable of satisfying these conditions. Indeed with a defect of a diameter of the order of 1 μm and and selected atomic velocities [13,14], around 300 m/s, $\bar{\tau}_1$ and $\bar{\tau}_2$ are of the order of 10^{-8} s. The lifetime of a photon in the strongly localized mode of the defect is of the order of 0.1–1 ms [22] which for micrometre sized defects corresponds to quality factors of the order of 10^{10}–10^{11}. These kind of quality factors should be reachable for the case of empty void regions embedded in a high quality dielectric (this even extends to the case when 10% of the mode lies in the dielectric [19,22]). Hence it suffices to choose the rate of emission from oven 1 so that T_1 is of the order of 10^{-2} s, and T_2 is of the order of 10^{-6} s, for the above conditions to be satisfied. In contrast to microwave cavity proposals [12–14], the data accumulation rate here is much faster.

Secondly, as it is impossible to tune the atomic velocities with infinite precision, the atoms entering the crystal will possess a spread in their velocities thus affecting the interaction times. One could argue that this could average out the strength of the correlations described above resulting to no net violation. However the violation of the Bell inequality in our scheme is still quite strong even in the case of large spreads in the atomic velocities. As a model we assume a box distribution [12] where the atomic velocities follow a flat distribution of width Δv around some average velocity $\bar{v}$ (*i.e.* that from a chopper)

$$P_{\rm i}(v_{\rm i}) = \begin{cases} \frac{1}{2\Delta v} & \text{for} \qquad \bar{v}_{\rm i} - \Delta v \leq v_{\rm i} \leq \bar{v}_{\rm i} + \Delta v \\ 0 & \text{otherwise} \end{cases} . \tag{12}$$

As shown earlier the Bell inequality is maximally violated for the singlet state, equation (10). It occurs for the velocities $v_1 = 231$ m/s and $v_2 = 270$ m/s. Choosing these as the average values $\bar{v}_{\rm i}$ for the corresponding velocity spreads, the idealized expectation value $\mathcal{S}$, equation (11) becomes

$$\bar{\mathcal{S}} = \iint P_1(v_1) P_2(v_2) \mathcal{S}(v_1, v_2)\, {\rm d}v_1 {\rm d}v_2. \tag{13}$$

In Figure 4 we plot $\bar{\mathcal{S}}$ as a function of Δv for the case of $\bar{v}_1 = 231$ m/s and $\bar{v}_2 = 270$ m/s. We see a violation of Bell's inequality as long as $\Delta v \leq 35$ m/s holds. Using current atomic velocity selection technology, precisions [13,14] of the order of ± 2 m/s can be achieved.

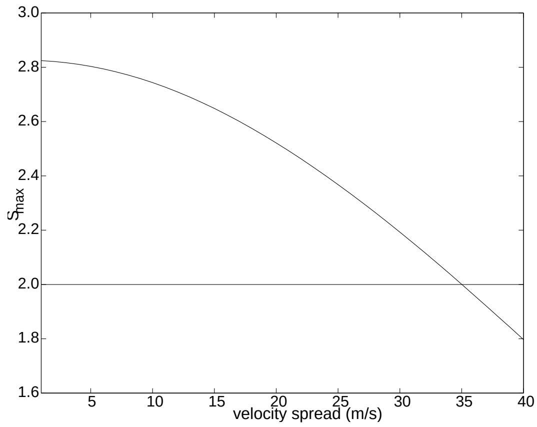

Fig. 4. The robustness of the scheme as a function of the spread in the atomic velocities. The vertical axis is the value the quantum mechanical sum $\bar{\mathcal{S}}$, equation (13) and the horizontal the atomic velocity spread Δv. A violation of Bell's inequality is predicted as long as $\Delta v \leq 34$ m/s.

In that case we find the quantum mechanical prediction $\bar{\mathcal{S}} \approx 2.812$, is almost as big as the maximal violation of the Bell inequality for the pure singlet state!

To complete the description of the potential of this scheme as a loophole free way of testing the Bell Inequalities, we have also to calculate the modification of the strength of the violation due the detector inefficiencies.

Defining the efficiencies η_A, η_B as the ratio of *detected* events of a certain kind to the *ideally detected* ones of that kind, it can be shown [12] that local realistic theories will not be possible if $(\eta_A + \eta_B - 1)\bar{\mathcal{S}} > 2$. For this to hold, $\eta = \eta_A = \eta_B$ must be larger than 0.855 as $\bar{\mathcal{S}}$ has the value 2.812. For a clear violation, an efficiency η of 0.9 or better is needed. In our case, as we need to distinguish between optically separated transitions this kind of efficiency is feasible [13,14].

Lastly we provide a qualitative argument for the potential closure of the communication loophole in our scheme. The lifetime of our entangled state is of the order of seconds and the atoms separate with a velocity of a few hundred m/s.

The analyzing could be done by Raman process (as we have a dipole-non allowed transition) through a virtual fourth level. The speed of that process can be as high nanoseconds or ten of nanoseconds. To this we have to add the detection time which although it varies with the detection process used, can be as small as 10^{-8} s (for ultrafast ionization or shelving techniques for example). This means that the duration of the whole process is bounded by 10 ns which is one or two order of magnitude less than the time needed for a light signal to cover the distance between the two atoms. No subluminal communication should thus be possible as long as the atoms are separated by a few meters.

In summary we propose a loophole free Bell inequality experiment where atoms are entangled by the interaction with a highly localized defect mode in an optical photonic crystal. The use of optically separated transitions which are stable to background radiation and can be detected with almost 100% efficiency can lead to the closure of the detection loophole. In the calculation of the strength of the correlations, we used a statistical operator approach. This includes the effect of finite precision in the velocity of the atoms in the beam. We showed that even for inaccuracies of order of a few dozen metres per second, the violation of the Bell inequality was still quite strong. Our system is capable of satisfying the appropriate time conditions imposed by the lack of simultaneity in the generation of our entangled particles as in the photon type experiments. In addition the relatively long lifetime of our final entangled state could allow for the solution of the communication loophole.

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